Managerial Incentives and the Specification of Functional Forms*

I. Introduction

A number of years ago Baumol [3] argued that corporate behavior in an imperfectly competitive world is represented not by profit maximization but instead by maximization of sales subject to a minimum profits constraint. Baumol's assertion has led to a fairly lengthy empirical literature that tries to determine the dominant objective behind firm behavior. However, a consensus has not been reached.[1] Depending on which study one reads one may conclude that the primary objective of the firm is sales but not profits, profits but not sales, or both profits and sales.

A primary source of the contrasting results may stem from the manner in which Baumol's assertion has been examined. The empirical literature has tested the assertion by first assuming that the top executive of a firm receives remuneration in accordance with his success in directing the corporation toward its goal. A reduced form regression equation is specified with profits and sales being the explanatory variables for executive compensation. Then, if executive compensation is found to be a function of profits but not sales it is concluded that the profit maximization hypothesis is supported. On the other hand, if sales is the only statistically significant variable, then the sales maximization hypothesis is concluded to be dominant.

Since this empirical approach allows a variety of functional forms to be specified for the reduced form equation that relates executive compensation to sales and profits, a possibility for conflicting results arises. The use of different functional specifications has not been based on theory, or on an empirical search to find the "best" specification. Rather, the functional specification has been chosen for a variety of *ad hoc* reasons.[2] This approach is

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1. Roberts [15], McGuire, Chiu and Elbing [13], Lewellen [9], Yarrow [17], and Ciscel [5] claimed sales to be the primary goal; Lewellen and Huntsman [10], Larner [8], Masson [11], and McEachern [12] support profit; and Cox and Shauger [6], Smyth, Boyes and Peseau [16] found both sales and profit to be objectives of firms.

2. Ciscel [5], Roberts [15], McGuire, Chiu and Elbing [13] and Larner [8] used a linear form while Cox and Shauger [6] and Yarrow [17] used the log-linear form seemingly on the basis of

subject to specification errors. Misspecification can lead to inconsistent and/or biased estimates as well as to errors in judging the relative importance of profits and sales in determining executive remuneration. The purpose of this paper is to examine functional specifications in the relation between sales, profits and executive remuneration. The approach we take is to estimate a general functional form, that is, one that includes as special cases the forms usually estimated. Then, this general functional form is examined for the presence of specification errors.

II. Specification of the Functional Forms

Executive remuneration (ER) is specified to depend on sales (S) and profits (P):

$$ER = ER(S,P) \tag{1}$$

The specific functional forms of Equation (1) that are considered for estimation purposes are members of the class of Box and Cox [4] transformations. These transformations are employed so that the parameters describing the form of Equation (1) may be estimated rather than imposed on an *ad hoc* basis. Specifically, in the Box-Cox model, the equation to be estimated is assumed to be linear in variables transformed according to:

$$Z^{(\lambda)} = \begin{cases} (Z^{\lambda} - 1)/\lambda & \lambda \neq 0 \\ \ln Z & \lambda = 0 \end{cases} \tag{2}$$

Thus, while the Box-Cox family is not exhaustive, it does incorporate many transformations—such as linear, reciprocal, logarithmic, semi-logarithmic, and polynomial—that are commonly used in applied econometric research.

The most general specification of Equation (1) that is permitted within the Box-Cox class is

$$ER^{(\lambda_0)} = \alpha_0 + \alpha_1 S^{(\lambda_1)} + \alpha_2 P^{(\lambda_2)} + \mu \tag{3}$$

where each variable may be subjected to a different transformation. In addition, in Equation (3), a disturbance term (whose properties will be discussed below) has been included. This equation, which will be referred to as the Generalized Box-Cox (GBC), has not been used in the empirical works cited

estimation ease. In other studies, the functional form was influenced by an attempt to minimize some statistical problem. For example, Lewellen and Huntsman [10] and Smyth, Boyes and Peseau [16] divided the linear form by a firm size measure to minimize heteroscedasticity problems. Only Baker [2] specifically attempted to choose a "best" functional form. He compared linear, log-linear and semi-log forms using correlation coefficients. However, because the dependent variables were different, Baker's comparison based on correlation coefficients were faulty. See Smyth, Boyes, and Peseau [16, 72–3].

above. However, if $\lambda_0 = \lambda_1 = \lambda_2 = 0$ the GBC from reduces to the log linear model estimated by Yarrow [17] and Cox and Shauger [6].

$$\ln ER = \alpha_0 + \alpha_1 \ln S + \alpha_2 \ln P + \mu \tag{4}$$

Also, if $\lambda_0 = \lambda_1 = \lambda_2 = 1$, the GBC model reduces to the linear model used by most researchers.

$$ER = \alpha_0 + \alpha_1 S + \alpha_2 P + \mu \tag{5}$$

The use of the Generalized Box-Cox approach has some interesting implications for the estimation of elasticities. The *a priori* choice of a functional form restricts the behavior of sales and profit elasticities. In the log-linear model the elasticities are constant, while in the linear model the elasticities converge toward one. In either case, the tradeoff between sales and profit is constant. In the GBC model the elasticities are the result of estimation, not *a priori* choice.

III. Estimation of Models

In order to obtain estimates of the parameters in the four models, assume that for some (unknown) triplet λ_0, λ_1, λ_2, the disturbance term, μ, in Equation (3) is normally, independently and identically distributed with zero mean and variance σ^2. Under these assumptions, maximum likelihood estimates for the GBC model may be found by maximizing the log-likelihood function:

$$\begin{aligned} L = \text{constant} + \log J - (n/2)\log\sigma^2 - (n/\sigma^2)\{ER^{(\lambda_0)} - \alpha_0 \\ - \alpha_1 S^{(\lambda_1)} - \alpha_2 P^{(\lambda_2)}\}^2 \end{aligned} \tag{6}$$

where $\log J = (\lambda_0 - 1)\sum \log ER$ and where J is the determinant of the Jacobian of the transformation of the μ's to the ER's. Partially differentiating (6) with respect to α_0, α_1, α_2, and σ^2 and setting these derivatives equal to zero yields the familiar maximum likelihood estimates for these parameters in terms of λ_j, $j = 0, 1, 2$. However, since the λ_j are unknown, it is necessary to search over a range of admissable values for these parameters in order to find the combination $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$, $\hat{\lambda}_0$, $\hat{\lambda}_1$, $\hat{\lambda}_2$ that maximizes the concentrated likelihood function:

$$L_c = \text{constant} + (\hat{\lambda}_0 - 1)\sum \log ER - (n/2)\log\hat{\sigma}^2 \tag{7}$$

We apply this approach to data on 782 firms for the year 1976 as reported in the *Forbes Annual Directory* [1]. Executive remuneration is measured in dollars[3] and sales and profits are expressed in thousands of dollars.[4]

3. The measure of executive remuneration is called "Total Remuneration" in *Forbes*. It includes salary, bonuses (including bonuses paid in unrestricted shares of company stock), director's fees and deferred compensation.

4. Other influences on remuneration than profits or sales have been considered in a few

The estimates are presented in Tables I and II. In interpreting these results, it should be recalled that they were obtained by a search procedure that involved systematically varying the λ_j parameters. Consequently, the estimates presented for the GBC model refers to the iteration that had the highest concentrated likelihood value.[5] In addition, the estimates for the linear and loglinear models are presented.

In Table I approximate confidence intervals at the 99% significance level for the estimates of the λ_j in the GBC model are also presented. These intervals are obtained from the result (Kendall and Stuart [7]) that the statistic $-2[L_{max}(\lambda_j^0) - L_{max}(\hat{\lambda}_j)]$ is distributed as χ^2 with one degree of freedom where $L_{max}(\hat{\lambda}_j)$ denotes the value of the unconditional maximum of the concentrated likelihood function and $L_{max}(\lambda_j^0)$ denotes the value of the conditional maximum of L_c with λ_j constrained to equal $\hat{\lambda}_j^0$.[6] The confidence bounds each include the point 0.0 but not 1.0. A joint confidence bound of all the λ_j can be derived in a similar manner.

Since the models are members of the same family of parametric functions, a likelihood ratio test can be used to determine whether the "goodness of fit" of the linear and log-linear models are significantly different from the GBC model. This test is again based on the Chi-squared distribution. The results of these are presented in the pairwise comparisons shown in Table II. The GBC model is unrestricted, the linear model and log-linear model have the restrictions $\lambda_0 = \lambda_1 = \lambda_2 = 0.0$ and $\lambda_0 = \lambda_1 = \lambda_2 = 1.0$ respectively.

The results show a significant (at more than the .999 level) difference between the linear model and the unrestricted GBC model. However, even though the GBC model has a higher calculated likelihood, the difference between it and the log-linear model is not significant at standard levels.

It should be noted however, that although the GBC and log-linear models are significantly different only at very low levels of significance, the behavioral implications of the two models are somewhat different. For example, the elasticity of substitution between sales and profits is $-.36(P^{1.049}/S^{1.089})$ for the GBC model and is $-.32$ for the log-linear model. If profits and sales are approximately the same then the elasticity for the two models are approximately the same. In most cases however, the elasticities for the two models will differ substantially. For example, at $P = 27,374$, $S = 628,743$ the elasticity of substitution is approximately $-.014$. Similarly, in the GBC model the effect

studies, e.g. value of shares (Masson [11]) or assets (Ciscel [5]). We did not consider these other influences because we wanted to focus on the contribution of estimating a general functional form in reconciling the results coming from studies using profits and sales only.

5. In the GBC model the λ_j were varied over the range .2 to 3.0 by steps of 0.1. Recall, the estimates for the linear model and the log linear models were those for which in the GBC model $\lambda_j = 0.0$ and $\lambda_j = 1.0$, $i = 0, 1, 2$ respectively.

6. It would seem to be more straight forward to obtain an asymptotic measure of the precision of these estimates from the information matrix associated with the likelihood function. However, as Zarembka [18] has previously reported, in the context of the Box-Cox model this approach is too complicated to be worth the cost; expectations of the non-linear functions of random variables are required.

Table I. Maximum Likelihood Estimates

Model	Constant[a]	Coefficient[a] on Sales	Coefficient[a] on Profits	Value of Transformation Parameter[b]	R^2
GBC	-.80315 (-3.7423)	.5029 (16.893)	.18808 (4.7016)	λ_0 = .000 (-.01 to .100) λ_1 = -.089 (-.2 to .09) λ_3 = -.049 (-.15 to .04)	.53
Linear	231.84 (38.0776)	.2126-04 (7.41)	.664-05 (.91)	λ_0 = 1.0[c] λ_1 = 1.0[c] λ_2 = 1.0[c]	.27
Log Linear	1.180 (8.0776)	.2565 (6.055)	.07495) (4.5369)	λ_0 = 0.0[c] λ_1 = 0.0[c] λ_2 = 0.0[c]	.526

Note: [a] t-values in parentheses below coefficients
[b] 99% confidence bounds
[c] fixed *a priori*

Table II. Pairwise Comparison of Commonly Used Functional Forms Using Likelihood Ratio Test

Models Compared	χ^2	(d.f.)	Probability of a larger value than χ^2
GBC - Linear	70.2	(3)	.0001
GBC - Log Linear	1.6	(3)	.60
Linear - Log Linear	68.3	(1)	.0001

on executive remuneration of a 1% increase in sales or profits will depend on the size (measured by profits or sales) of the firm., However, in the log-linear model, the elasticities are constant. A 1% increase in sales leads to a .25% increase in remuneration for the log-linear model whereas for the GBC model a 1% increase in sales leads to a $.5029/S^{.089}$ percent increase in remuneration.

These differences in behavioral implications and the results of the likelihood ratio tests suggest that we should subject the models to further scrutiny. Specifically, we are interested in whether the models are misspecified.

IV. Specification Error Tests

In this section the validity of the standard least squares assumptions regarding the properties of the disturbance term in each of the four models is tested via residual analysis. As is well known, these assumptions require that the disturbance terms must be identically and independently distributed with mean of zero and constant variance for all observations.

The validity of these assumptions is examined by considering the following four types of specification error: (1) omitted variables, (2) incorrect functional form, (3) simultaneity, and (4) heteroscedasticity. The first three types of errors, termed Group A errors, cause the assumption $E(\mu) = 0$ to be violated and lead to inconsistent estimates of the regression parameters, while the fourth type of error (Group B) changes the covariance matrix of the estimates of the slope coefficients. Tests for these groups of errors have been described in detail by Ramsey [14]. Consequently they will not be discussed at length here. However, it should be mentioned that the RESET test, which is an F-test, is used to test against Group A errors, and Bartlett's M statistic is used in an attempt to detect the presence of Group B errors. Also, it should be noted that the Group B errors (e.g., heteroscedasticity) have been at the center of much of the empirical work on managerial incentives, as shown in Lewellen and Huntsman [10] and Smyth, Boyes and Peseau [16].

The results of the specification error tests are presented in Table III. The tests were applied to the GBC, log-linear and linear models. The test for Group A errors, RESET, does not reject the null hypothesis for either GBC or log-linear; it does for the linear model. However, the test does show the log-linear model to be subject to more heteroscedasticity than the GBC model. Thus, the GBC model is supported by the specification error tests. The linear form does contain misspecifications. And while the log-linear form is not found to be misspecified, it is not as strongly supported as the GBC model.

Table III. Specification Error Tests

MODEL	RESET F	BAMSET M
Linear	3.8617 R (3,779)	18.1978 R_5R_{10} (2)
Log linear	2.4112 A (3,779)	1.8667 A_5A_{10} (2)
GBC	1.3875 A (3,779)	.4620 $A_1A_5A_{10}$ (2)

Note: Degrees of freedom in parenthesis
The subscript 5 or 10 on the A or R indicates at what level of the test the hypothesis was accepted or rejected. If no subscript is indicated the test was carried out at the 90 per cent level.

V. Conclusion

In summary, the "best" specification is the GBC model. However, since the GBC model is statistically close to a log-linear model, we should expect that the results regarding the significance of profit and sales in explaining the variance of executive compensation be close to those studies using a log-linear form. In the Baker [2] and Cox-Shauger [6] log-linear models both profits and sales were found to be significant. This is what our results in Table I show also, even though our sample is much larger than the other studies. Interestingly, comparing the GBC model to these log-linear models, we find profit to be less important than sales.[7] The linear model, in which sales was

7. A statistical problem which creates difficulties for judging the relative contributions of sales and profits is collinearity between the two independent variables. The simple correlation

generally found to be significant while profit was not, was found to be misspecified. Thus, by estimating a general functional form, we have been able to provide some information which helps reconcile the conflicting results existing in the literature.

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coefficient between sales and profits in our sample is .888. After the transformation the simple correlation coefficient is .717. This is significant (at the .01 level) reduction. However, .717 is still a significant correlation. Smyth, Boyes, and Peseau [16] found that dividing the linear form by a measure of firm size tended to reduce both heteroscedasticity and collinearity. We did not pursue that avenue here because we were primarily interested in the relation between executive remuneration and profits and sales no matter the firm size. That is, we did not want to attempt to explain the ratio of executive compensation to firm size. Rather, we wished to account for the variation in the level of executive remuneration.

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